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International Journal of Solids and Structures 41 (2004) 4685–4696

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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Periodic array of cracks in a strip subjected to surface heating

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Received 16 July 2003; received in revised form 25 February 2004
Available online 9 April 2004

Abstract

The analysis of a periodic array of cracks in an infinite strip under surface heating is investigated. The thermal stresses are generated as a result of a ramp function change on the boundary. Due to surface heating, compressive transient thermal stresses occur close to the surface causing the crack surfaces to come into contact at a certain contact length. The problem is treated as a nonlinear contact crack problem with the smooth closure condition of the crack surfaces. The mixed boundary value problem is reduced to a hypersingular integral equation with the crack surface displacement as an unknown function and the crack contact length as an additional unknown variable. Numerical results for stress intensity factors and the crack contact length are obtained as a function of time, heating rate, crack length, and periodic crack spacing.

Keywords: Thermal stresses; Stress intensity factor; Fracture mechanics; Periodic crack

1. Introduction

Many researchers have been concerned with the study of an elastic plate under thermal loading especially in the presence of preexisting cracks since it exists in many engineering applications such as aircraft, turbine engines, and power plants. The single crack problem in a finite width and a semi-infinite plate under thermal stresses has been investigated (see, for example, Nied, 1983, 1987; Rizk and Radwan, 1992; Rizk, 1993a). Some studies of multiple cracks in a plate under thermal loading are considered in the literature. Bahr et al. (1988) studied an array of parallel and equally spaced edge cracks in a long strip due to quenching using the boundary element method. Multiple crack problems in the functionally graded materials under thermal loading are examined by Wang et al. (2000). The problem of a periodic array of cracks in a half-plane subjected to convective cooling is considered by Rizk (2003).

In this paper the study of a periodic array of edge cracks perpendicular to the boundaries in an elastic infinite plate subjected to surface heating is considered. When the surface of the plate is heated, compressive stresses are generated near the plate surface forcing the crack surface to come into contact along a certain

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0020-7683/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/j.ijsolstr.2004.02.065

contact length. The computation of the stress intensity factors that does not take into account the crack contact length will lead to negative results. Nevertheless, if the crack contact length is considered in the analysis, the crack is cusp shaped and the smooth closure condition of the crack surfaces can be used leading to positive stress intensity factors at the crack tip. The crack contact length is therefore an additional unknown variable, which must be introduced into the formulation of the crack problem. So, the problem of interest would be a nonlinear contact problem, which is treated as an embedded crack problem with a smooth closure condition that must be solved iteratively.

In the analysis, it is assumed that the material is linear isotropic homogenous, whilst the thermoelastic coupling effects and the temperature dependence of the thermoelastic coefficients are negligible. The transient thermal stress problem may be treated as a quasi-static, that is, inertia effects are negligible. Previous studies on dynamic thermoelasticity seem to justify this assumption (Sternberg and Chakravorty, 1959a,b).

Since the material is linear, the principle of superposition can be used to solve the problem. The stress state in the strip may be considered as the sum of two solutions. The first is to obtain the transient thermal stresses in the strip without cracks. The second is to solve the isothermal cracked problem (the perturbation problem) by using the thermal stresses obtained from the first with an opposite sign on the crack surfaces as the only external loads. The superposition of the two solutions gives the results for the thermal stress problem for the cracked medium. Because our interest is in the stress intensity factor, it is sufficient to consider the perturbation problem only. It is important to note that the presence of the cracks that are normal to the face of the strip does not perturb the one-dimensional transient temperature and thermal stress distribution, which are in x direction. By expressing the displacement components in terms of finite and infinite Fourier transforms (Schulze and Erdogan, 1998), and defining a new function in terms of the crack surface displacement, we end up with a hypersingular integral equation which is solved numerically using the expansion method and the concept of the finite-part integral developed by Kaya and Erdogan (1987). Numerical calculations for stress intensity factors are obtained as a function of normalized quantities such as time, heating rate, crack length, and periodic crack spacing.

2. Mathematical formulation

Consider an elastic strip of thickness h with an infinite array of edge cracks at a distance $2c$ as shown in Fig. 1a. Assume the medium is initially at a constant temperature T_∞, and at $t = 0$ the surface $x = 0$ is

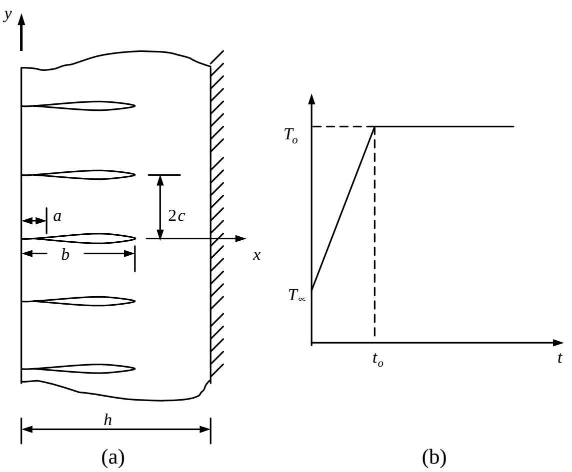

Fig. 1. (a) Geometry of periodic cracks, (b) ramp function on the boundary.

heated gradually to T_0 by a ramp function shown in Fig. 1b while the boundary $x = h$ is insulated. It is clear that the heating rate is decreases as t_0 increases. The most rapidly heating rate of change occurs when $t_0 = 0$ that is corresponding to a unit step function change. Since the solution of the temperature distribution as well as the transient thermal stresses are independent of whether the surface is heated or cooled, the transient thermal stresses for the uncracked problem that are needed to formulate the cracked medium can be obtained from Rizk (1993a,b). For a unit step function ($\tau_0 = 0$)

$$\frac{\sigma_{yy}^{\mathrm{T}}(x^*,\tau)(1-\upsilon)}{E\alpha(T_0-T_\infty)} = 2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}}{\lambda_n\sin\lambda_n}\cos\lambda_n(x^*-1) - (4-6x^*)\left[2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}}{\lambda_n^2}\right] - (12x^*-6)\left[2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}}{\lambda_n^3\sin\lambda_n}\right] \tag{1}$$

and for a ramp function ($\tau_0 > 0$)

$$\begin{aligned}\frac{\sigma_{yy}^{\mathrm{T}}(x^*,\tau)(1-\upsilon)}{E\alpha(T_0-T_\infty)} &= \left[\left(1-\frac{\tau}{\tau_0}\right) - 2\sum_{n=1}^{\infty}\frac{(\mathrm{e}^{-\tau\lambda_n^2}-1)}{\tau_0\lambda_n^3}\frac{\cos\lambda_n(x^*-1)}{\sin\lambda_n}\right] - (4-6x^*)\left[\left(1-\frac{\tau}{\tau_0}\right) - 2\sum_{n=1}^{\infty}\frac{(\mathrm{e}^{-\tau\lambda_n^2}-1)}{\tau_0\lambda_n^4}\right] \\ &\quad - (12x^*-6)\left[\frac{1}{2}\left(1-\frac{\tau}{\tau_0}\right) - 2\sum_{n=1}^{\infty}\frac{(\mathrm{e}^{-\tau\lambda_n^2}-1)}{\tau_0\lambda_n^5}\frac{1}{\sin\lambda_n}\right], \quad \tau \leqslant \tau_0\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\sigma_{yy}^{\mathrm{T}}(x^*,\tau)(1-\upsilon)}{E\alpha(T_0-T_\infty)} &= \left[2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}(\mathrm{e}^{-\tau_0\lambda_n^2}-1)}{\tau_0\lambda_n^3}\frac{\cos\lambda_n(x^*-1)}{\sin\lambda_n}\right] - (4-6x^*)\left[2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}(\mathrm{e}^{\tau_0\lambda_n^2}-1)}{\tau_0\lambda_n^4}\right] \\ &\quad - (12x^*-6)\left[2\sum_{n=1}^{\infty}\frac{\mathrm{e}^{-\tau\lambda_n^2}(\mathrm{e}^{\tau_0\lambda_n^2}-1)}{\tau_0\lambda_n^5}\frac{1}{\sin\lambda_n}\right], \quad \tau > \tau_0\end{aligned} \tag{3}$$

where E is the Young's modulus, υ is the Poisson's ratio, α is the coefficient of linear thermal expansion and D is the thermal diffusivity. Also $x^* = x/h$, $\tau_0 = t_0D/h^2$, $\tau = tD/h^2$ (Fourier number), and $\lambda_n = \pi/2(2n-1)$ are dimensionless parameters. Referring to Fig. 1a, the plane elasticity problem requires the solution of the following equilibrium equations:

$$(\kappa-1)\nabla^2 u + 2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x\partial y}\right) = 0 \tag{4}$$

$$(\kappa-1)\nabla^2 v + 2\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial y^2}\right) = 0 \tag{5}$$

where $\kappa = (3-4\upsilon)$ is valid for plane strain and $\kappa = (3-\upsilon)/(1+\upsilon)$ for plane stress, and u, v are the x and y components of the displacement vector. Because of periodicity, the problem was considered for $0 < y < c$ and subjected to the following homogenous boundary conditions:

$$\sigma_{xy}(x,0) = 0, \qquad \sigma_{xy}(x,c) = 0, \qquad v(x,c) = 0, \quad 0 < x < h \tag{6}$$

$$\sigma_{xx}(0,y) = 0, \qquad \sigma_{xy}(0,y) = 0, \qquad \sigma_{xx}(h,y) = 0, \qquad \sigma_{xy}(h,y) = 0, \quad 0 < y < c \tag{7}$$

and the mixed boundary condition

$$v(x,0) = 0, \quad 0 < x < a, \ b < x < h, \qquad \sigma_{yy}(x,0) = -\sigma_{yy}^{\mathrm{T}}(x,t), \quad a < x < b \tag{8}$$

where $\sigma_{ij}(i,j=x,y)$ are the stresses and $\sigma_{yy}^{\mathrm{T}}(x,t)$ is the thermal stress from the uncracked problem. The solution of the differential equations (4) and (5) can be obtained by assuming the displacement components in terms of sums of finite and infinite Fourier transforms (Schulze and Erdogan, 1998) in the form

$$u(x,y)=\sum_{n=0}^{\infty}f(x,\alpha_n)\cos\alpha_n y+\frac{1}{2\pi}\int_{-\infty}^{\infty}p(y,\beta)\mathrm{e}^{\mathrm{i}x\beta}\,\mathrm{d}\beta \tag{9}$$

$$v(x,y)=\sum_{n=1}^{\infty}g(x,\alpha_n)\sin\alpha_n y+\frac{1}{2\pi}\int_{-\infty}^{\infty}q(y,\beta)\mathrm{e}^{\mathrm{i}x\beta}\,\mathrm{d}\beta \tag{10}$$

where $\alpha_n=\pi n/c$. By a substitution of Eqs. (9) and (10) into the Eqs. (4) and (5) it can be shown that the displacement components u, v have the following form

$$u(x,y)=\sum_{n=0}^{\infty}[(C_1+C_2x)\mathrm{e}^{x\alpha_n}+(C_3+C_4x)\mathrm{e}^{-x\alpha_n}]\cos\alpha_n y+\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{i|\beta|}{\beta}\left[\left(D_1+\frac{\kappa}{|\beta|}D_2+D_2y\right)\mathrm{e}^{y|\beta|}\right.$$
$$\left.-\left(D_3-\frac{\kappa}{|\beta|}D_4+D_4y\right)\mathrm{e}^{-y|\beta|}\right]\mathrm{e}^{\mathrm{i}x\beta}\,\mathrm{d}\beta \tag{11}$$

$$v(x,y)=\sum_{n=1}^{\infty}\left[\left(-C_1-\frac{\kappa}{\alpha_n}C_2-C_2x\right)\mathrm{e}^{x\alpha_n}+\left(C_3-\frac{\kappa}{\alpha_n}C_4+C_4x\right)\mathrm{e}^{-x\alpha_n}\right]\sin\alpha_n y$$
$$+\frac{1}{2\pi}\int_{-\infty}^{\infty}[(D_1+D_2y)\mathrm{e}^{y|\beta|}+(D_3+D_4y)\mathrm{e}^{-y|\beta|}]\mathrm{e}^{\mathrm{i}x\beta}\,\mathrm{d}\beta \tag{12}$$

where $C_i(i=1,2,3,4)$ are functions of α_n and $D_i(i=1,2,3,4)$ are functions of β. Seven of the unknown functions $C_i(\alpha_n)$, $D_i(\beta)$, $(i=1,\dots,4)$ can be eliminated by using the homogenous boundary conditions (6) and (7). The remaining one would then be given by the mixed boundary condition (8). By defining a new function as

$$\varphi(x)=v(x,0) \tag{13}$$

all the unknown functions can be expressed in terms of $\varphi(x)$. After a lengthy but straightforward analysis the problem will be reduced into the following integral equation with $\varphi(x)$ as the unknown function i.e.

$$\lim_{y\to 0}\int_a^b[K_1(x,y,s)+K_2(x,y,s)]\varphi(s)\,\mathrm{d}s=-\frac{\pi(\kappa+1)}{4\mu}\sigma_{yy}^{\mathrm{T}}(x,t) \tag{14}$$

where μ is the shear modulus, and the kernels K_1 and K_2 are given by

$$K_1(x,y,s)=\int_0^{\infty}\left\{\frac{[(1-y\beta)\mathrm{e}^{-4c\beta}+(y\beta-2c\beta-1)\mathrm{e}^{-2c\beta}]\mathrm{e}^{y\beta}}{(1-\mathrm{e}^{-2c\beta})^2}\right.$$
$$\left.-\frac{[(2c\beta-y\beta-1)\mathrm{e}^{-2c\beta}+(y\beta+1)]\mathrm{e}^{-y\beta}}{(1-\mathrm{e}^{-2c\beta})^2}\right\}\beta\cos(s-x)\beta\,\mathrm{d}\beta \tag{15}$$

$$K_2(x,y,s)=\frac{\pi}{c}\sum_{n=1}^{\infty}\left\{\left[-H_1-\left(\alpha_n x+\frac{\kappa+3}{2}\right)\frac{H_2}{M}\right]\mathrm{e}^{\alpha_n x}+\left[H_3+\left(\alpha_n x-\frac{\kappa+3}{2}\right)\frac{H_4}{M}\right]\mathrm{e}^{-\alpha_n x}\right\}\cos\alpha_n y \tag{16}$$

where H_i $(i = 1, 2, 3, 4)$, M are given by

$$H_1 = -\frac{1}{2}\alpha_n e^{(s-2h)\alpha_n} - \frac{1}{2}(\kappa + 2\alpha_n h)\frac{H_2}{M} + \frac{1}{2}e^{-2\alpha_n h}\frac{H_4}{M} \tag{17}$$

$$H_2 = [-(2s\alpha_n - 3)(2\alpha_n^2 h) - \alpha_n]e^{-(s+2h)\alpha_n} + [-2\alpha_n^2(s-h) - 3\alpha_n]e^{(s-2h)\alpha_n} + [2\alpha_n^2 h + 2\alpha_n^2(s-h) + 3\alpha_n]e^{(s-4h)\alpha_n} + [\alpha_n]e^{-(s+4h)\alpha_n} \tag{18}$$

$$H_3 = -\frac{1}{2}[2\alpha_n^2(s-h) + 3\alpha_n]e^{s\alpha_n} - \frac{1}{2}e^{2\alpha_n h}\frac{H_2}{M} + \frac{1}{2}(\kappa - 2\alpha_n h)\frac{H_4}{M} \tag{19}$$

$$H_4 = [2s\alpha_n^2 - 3\alpha_n]e^{-s\alpha_n} + [-\alpha_n + 2\alpha_n^2 h\{2\alpha_n(s-h) + 3\}]e^{(s-2h)\alpha_n} + [-\alpha_n(2s\alpha_n - 3) + 2\alpha_n^2 h]e^{-(s+2h)\alpha_n} + [\alpha_n]e^{(s-4h)\alpha_n} \tag{20}$$

$$M = 1 - (4\alpha_n^2 h^2 + 2)e^{-2\alpha_n h} + e^{-4\alpha_n h} \tag{21}$$

By separating the singular terms from K_1 and K_2 we may have

$$\lim_{y\to 0} K_1(x, y, s) = \frac{1}{(s-x)^2} + k_1^f(x, s) \tag{22}$$

$$\lim_{y\to 0} K_2(x, y, s) = k_{2a}^s(x, s) + k_{2b}^s(x, s) + k_2^f(x, s) \tag{23}$$

where $k_{2a}^s(x, s)$ and $k_{2b}^s(x, s)$ are the standard generalized singular terms at $a = 0$ and $b = h$ respectively, that are given by

$$k_{2a}^s = \frac{-1}{(s+x)^2} + \frac{12x}{(s+x)^3} - \frac{12x^2}{(s+x)^4} \tag{24}$$

$$k_{2b}^s = \frac{-1}{(2h-s-x)^2} + \frac{12(h-x)}{(2h-s-x)^3} - \frac{12(h-x)^2}{(2h-s-x)^4} \tag{25}$$

The kernel $k_1^f(x, s)$ is bounded as $(s - x) \to 0$ and it is in the form

$$k_1^f(x, s) = \int_a^b \left[\frac{2e^{-4c\beta} - 2(2c\beta + 1)e^{-2c\beta}}{(1 - e^{-2c\beta})^2}\right]\beta\cos(s-x)\beta\, d\beta \tag{26}$$

whereas, the kernel $k_2^f(x, s)$ is bounded as the crack approaches the free boundaries ($a = 0$ and $b = h$). So, the integral equation (14) may be expressed as

$$\int_a^b \frac{\varphi(s)}{(s-x)^2}\, ds + \int_a^b [k_{2a}^s(x, s) + k_{2b}^s(x, s) + k_1^f(x, s) + k_2^f(x, s)]\varphi(s)\, ds = -\frac{\pi(\kappa+1)}{4\mu}\sigma_{yy}^{T}(x, t) \tag{27}$$

Using the function theoretic method developed by Muskhelishvili (1953), and the finite part-integral described by Kaya and Erdogan (1987), the unknown function $\varphi(s)$ may be expressed as:

$$\varphi(s) = f(s)(s-a)^{\gamma_1}(b-s)^{\gamma_2} \tag{28}$$

where $f(s)$ is a continuous function in the interval $a \leqslant s \leqslant b$, $f(a) \neq 0$, $f(b) \neq 0$, and γ_1, γ_2 are depending on the location of the crack tip. If the crack is imbedded ($a > 0$, $b < h$) then $\gamma_1 = \gamma_2 = 1/2$, but for an edge crack ($a = 0$, $b < h$) $\gamma_1 = 0$, $\gamma_2 = 1/2$ and for ($a > 0$, $b = h$) $\gamma_1 = 1/2$, $\gamma_2 = 0$.

In the case of mode I, the stress intensity factor at the end points a and b is defined by

$$K(a) = \lim_{x \to a} \sqrt{2(a-x)}\sigma_{yy}(x,0) = \frac{4\mu}{\kappa+1} \lim_{x \to a} \frac{v(x,0)}{\sqrt{2(x-a)}} \tag{29}$$

$$K(b) = \lim_{x \to b} \sqrt{2(x-b)}\sigma_{yy}(x,0) = \frac{4\mu}{\kappa+1} \lim_{x \to b} \frac{v(x,0)}{\sqrt{2(b-x)}} \tag{30}$$

where $\sigma_{yy}(x,0)$ is the stress outside the crack. Following Muskhelishvili (1953), the stress intensity factor for an impeded crack at a and b are given by

$$K(a) = \frac{4\mu}{\kappa+1} f(a)\sqrt{\frac{b-a}{2}} \tag{31}$$

$$K(b) = \frac{4\mu}{\kappa+1} f(b)\sqrt{\frac{b-a}{2}} \tag{32}$$

In order to analyze the crack contact problem properly, the contact length, ε, in the compressive zone has to be considered as an additional unknown variable. The contact length may be calculated by using the smooth closure condition of the crack surface at $x = a$ which is assured by the condition $K(a) = 0$ (Bakioglu et al., 1976). So, the problem may be formulated as an impeded crack by fixing the crack length at $x = b$ and then determining by iteration the location of the crack tip $x = a$ at each time step so that the condition $K(a) = 0$ is satisfied.

3. Numerical technique

The numerical solution of Eq. (27) can be obtained by reducing it into a system of algebraic equations (Kaya and Erdogan, 1987). First, it is expressed in terms of the following normalized quantities

$$x = \frac{b-a}{2}r + \frac{b+a}{2}, \quad s = \frac{b-a}{2}\rho + \frac{b+a}{2} \tag{33}$$

$$f(s) = \left(\frac{b-a}{2}\right)^{\gamma_1+\gamma_2} F(\rho), \quad k(x,s) = \left(\frac{2}{b-a}\right)^2 h(r,\rho) \tag{34}$$

giving

$$\int_{-1}^{+1} \frac{F(\rho)(1+\rho)^{\gamma_1}(1-\rho)^{\gamma_2}}{(\rho-r)^2}\,\mathrm{d}\rho + \int_{-1}^{+1} h(r,\rho)F(\rho)(1+\rho)^{\gamma_1}(1-\rho)^{\gamma_2}\,\mathrm{d}\rho = -\pi\frac{\sigma_{yy}^{\mathrm{T}}(r,t)}{\sigma_0^{\mathrm{T}}} \tag{35}$$

where $\sigma_0^{\mathrm{T}} = E\alpha(T_0 - T_\infty)/(1-\upsilon)$. Assume the solution is in the form of a finite series as

$$F(\rho) = \sum_{n=0}^{N} a_n \rho^n \tag{36}$$

where a_n are $(N+1)$ unknown coefficients to be determined. By substituting Eq. (36) into Eq. (35) we end up with $(N+1)$ linear equations that are solved at certain collocation points. The zeros of Chebychev polynomials of the order $(N+1)$ are used as collocation points which are symmetrically distributed with respect to the origin and concentrated near the end points, i.e.

$$r_j = \cos\frac{\pi(2j-1)}{2(N+1)}, \quad j = 1,2,\ldots,N+1 \tag{37}$$

Finally the linear system of equations may be in the form

$$\sum_{n=0}^{N} a_n E^n(r_j) = -\pi \frac{\sigma^{\mathrm{T}}(r_j, t)}{\sigma_0^{\mathrm{T}}}, \quad j = 1, 2, 3, \ldots, N+1 \tag{38}$$

where

$$E^n(r_j) = \int_{-1}^{+1} \frac{\rho^n (1+\rho)^{\gamma_1} (1-\rho)^{\gamma_2}}{(\rho - r_j)^2} \mathrm{d}\rho + \int_{-1}^{+1} h(r_j, \rho) \rho^n (1+\rho)^{\gamma_1} (1-\rho)^{\gamma_2} \, \mathrm{d}\rho \tag{39}$$

The numerical integration techniques that are used are developed in Kaya and Erdogan (1987).

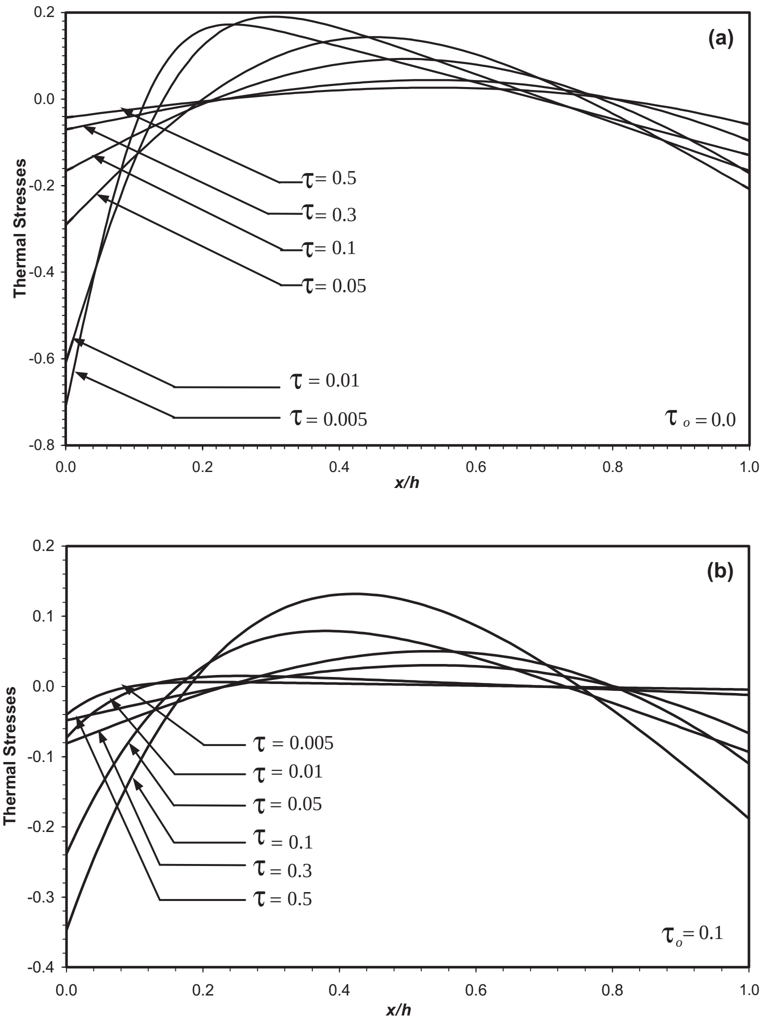

Fig. 2. Normalized thermal stresses $\sigma_{yy}^{\mathrm{T}}(x^*, \tau)/\sigma_0^{\mathrm{T}}$ for (a) unit step function $\tau_0 = 0$, (b) ramp function $\tau_0 = 0.1$.

4. Results and conclusion

Sample results of the thermal stresses $\sigma_{yy}^{\mathrm{T}}(x^*, \tau)/\sigma_0^{\mathrm{T}}$ are shown in Fig. 2. Fig. 2a is related to a unit step change at the boundary (Eq. (1)), while Fig. 2b is related to a ramp function change on the surface (Eqs. (2) and (3)). It is clear that as the heating rate decreases (τ_0 increases) the transient thermal stresses decrease consequently.

The influence of the normalized crack spacing $b/2c$ on the normalized stress intensity factors defined by $K(b)/\sigma_0^{\mathrm{T}}\sqrt{b}$ is shown in Fig. 3 as a function of the normalized time (Fourier number) $\tau = tD/h^2$ for two values of the normalized crack length $b/h = 0.2, 0.4$ and two values of the duration ramp $\tau_0 = 0.0$ and 0.1. Fig. 3a corresponds to the unit step function change $\tau_0 = 0.0$, while Fig. 3b corresponds to a ramp function change $\tau_0 = 0.1$. It can be seen that as normalized crack spacing increases (c decreases) the normalized stress intensity factors decrease. Thus, the maximum value of the normalized transient stress intensity factors will occur for a single crack ($c \to \infty$). Also by increasing the duration ramp τ_0 (heating rate

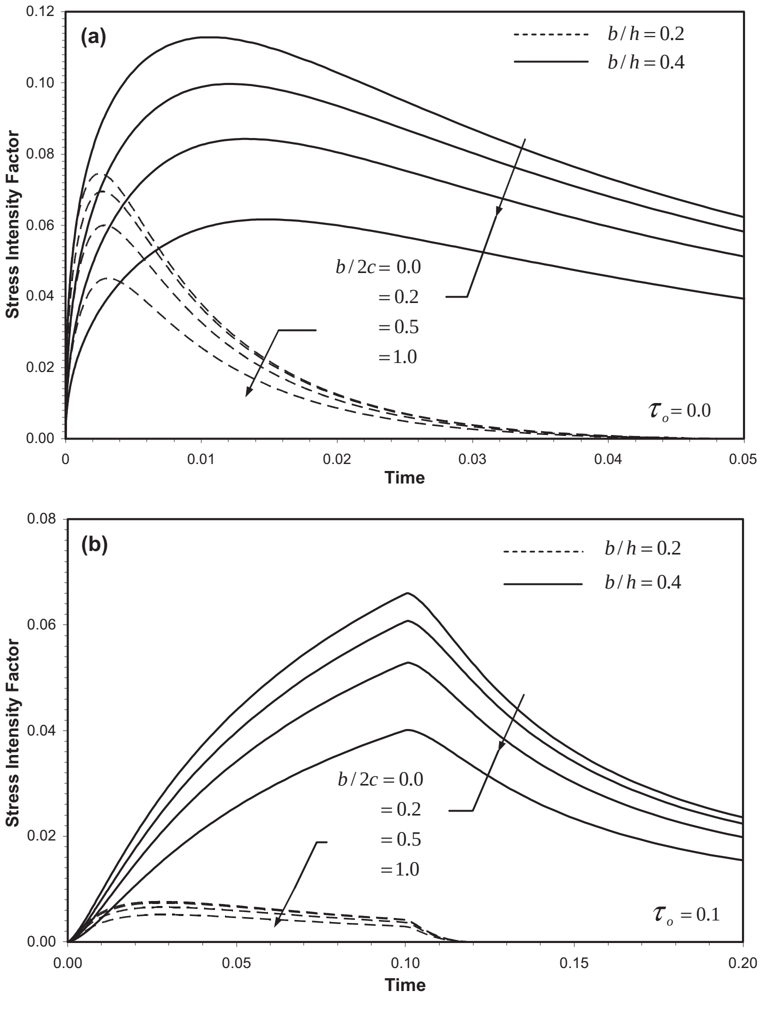

Fig. 3. Normalized stress intensity factor $K(b)/\sigma_0^{\mathrm{T}}\sqrt{b}$ for $b/h = 0.2, 0.4$. (a) Unit step function $\tau_0 = 0.0$, (b) ramp function $\tau_0 = 0.1$.

decreasing) the normalized stress intensity factors decrease. For all cases the normalized stress intensity factor increases as a function of time to a maximum value and then decreases rapidly for all values of $b/2c$ and τ_0. Fig. 4 demonstrate the normalized crack contact length ε/b versus the normalized time $\tau = tD/h^2$ for different values of $b/2c$, τ_0 and b/h. Fig. 4a and b are illustrated for $\tau_0 = 0.0$ and 0.1 respectively. It is obvious that as the normalized crack spacing increases (c decreasing), the crack contact length will increase. It should be noted that the location of the contact point at $x = a$ which satisfies the condition $K(a) = 0$ does not coincide with the point where the thermal stresses in the uncracked strip change sign. The contact point is located closer to the free surface $x = a$ than the zero in the stress field. For example, with $\frac{b}{h} = 0.4$, $\tau_0 = 0.0$, $\frac{b}{2c} = 1.0$ and $\tau = 0.005, 0.01, 0.05, 0.1$, the contact lengths relative to h are $\frac{\varepsilon}{h} = 0.093, 0.118, 0.179, 0.1985$, while the zero of the thermal stresses is located at $\frac{x}{h} = 0.115, 0.145, 0.205, 0.215$.

The effect of the crack length b/h on the normalized stress intensity factors versus the normalized time is shown in Fig. 5 for two values of $\tau_0 = 0.0, 0.1$, and two values of $b/2c = 0.1, 0.5$. Fig. 5a and b are depicted for $b/2c = 0.1$ and $b/2c = 0.5$ respectively. As the crack length increases the maximum value of

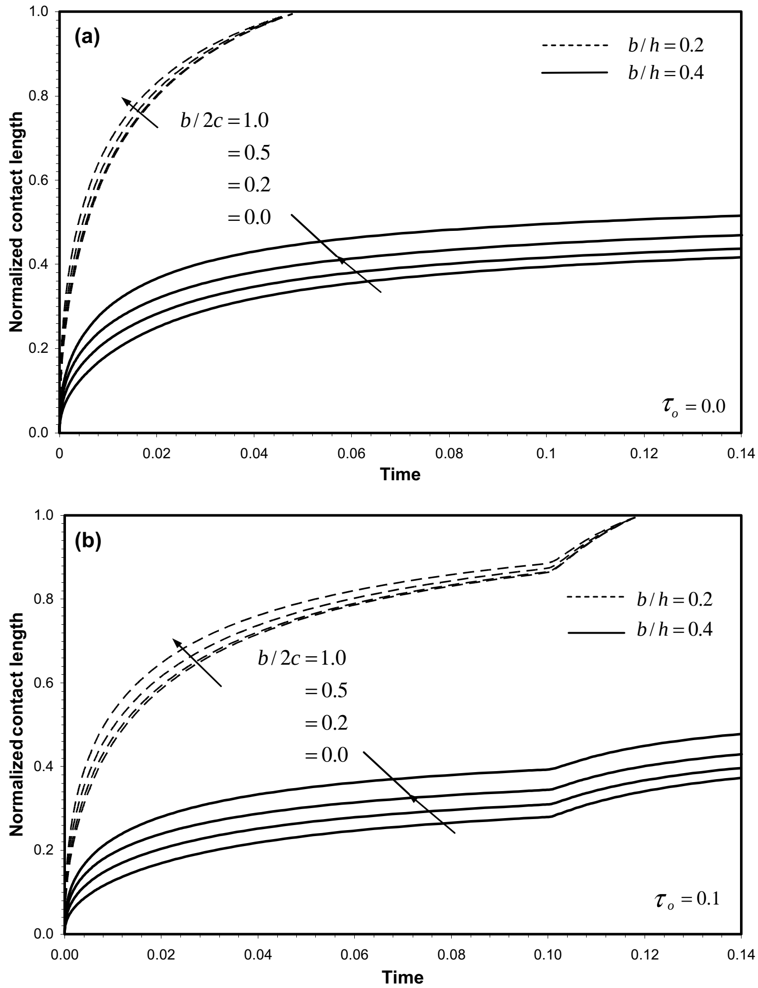

Fig. 4. Normalized crack contact length ε/b for $b/h = 0.2, 0.4$. (a) Unit step function $\tau_0 = 0.0$, (b) ramp function $\tau_0 = 0.1$.

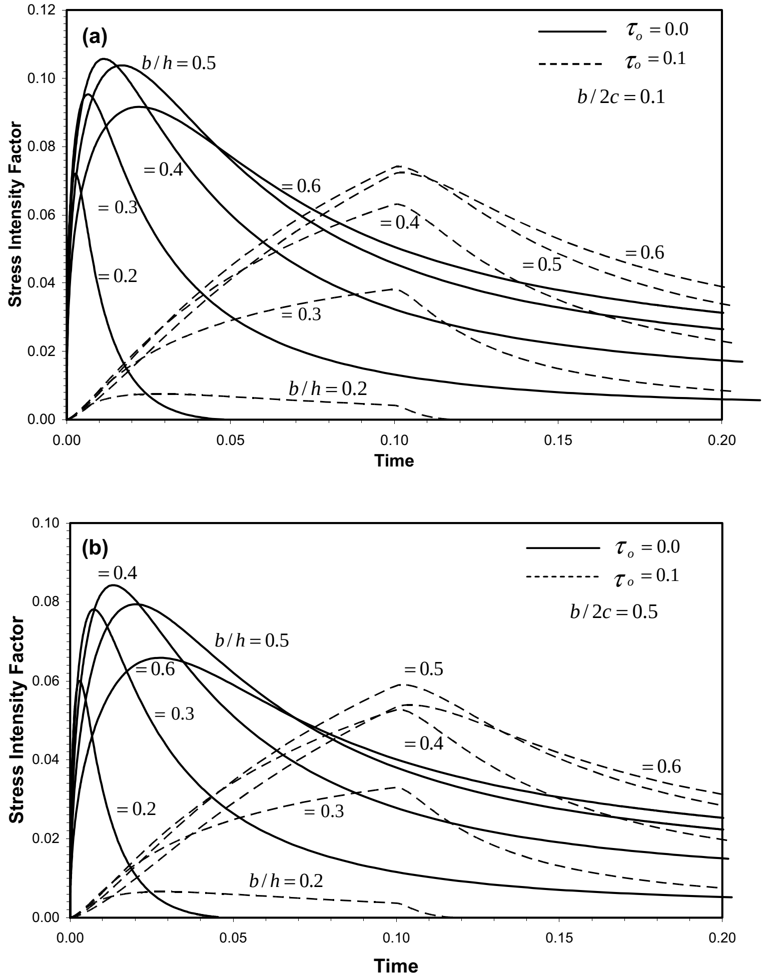

Fig. 5. Normalized stress intensity factor $K(b)/\sigma_0^{\mathrm{T}}\sqrt{b}$ for different values of normalized crack length b/h. (a) $b/2c = 0.1$, (b) $b/2c = 0.5$.

the normalized stress intensity factor is increases until a certain value of the crack length and then decreases for both cases $b/2c = 0.1, 0.5$. The influence of the heating rate measured by τ_0 and the normalized crack spacing $b/2c$ is also observed in the same figure. The normalized crack contact length ε/b is plotted against the normalized time for different values of the crack length b/h, two values of $\tau_0 = 0.0, 0.1$, and two values of $b/2c = 0.1, 0.5$; that is given in Fig. 6. Fig. 6a and b are related to $b/2c = 0.1$ and 0.5 respectively. It is apparent that, for the short crack $b/h = 0.2$, a complete crack closure over the entire length of the crack will occur. It is also clear that the normalized time needed to reach the maximum value of the normalized contact length increases as τ_0 increases. Obviously, the smallest magnitude of the normalized stress intensity factors will occur as the normalized periodic crack spacing $b/2c$ increases (c decreases).

In conclusion, the periodic crack spacing seems to be an important factor on the normalized stress intensity factors. As the distance between the crack spacing decreases, the magnitude of the normalized stress intensity factor will be reduced, and the normalized crack contact length would be increased due to the interaction between the cracks. Also, more realistic temperature change on the boundary (ramp function) gives more realistic results than the unit step function resulting in reducing the normalized stress

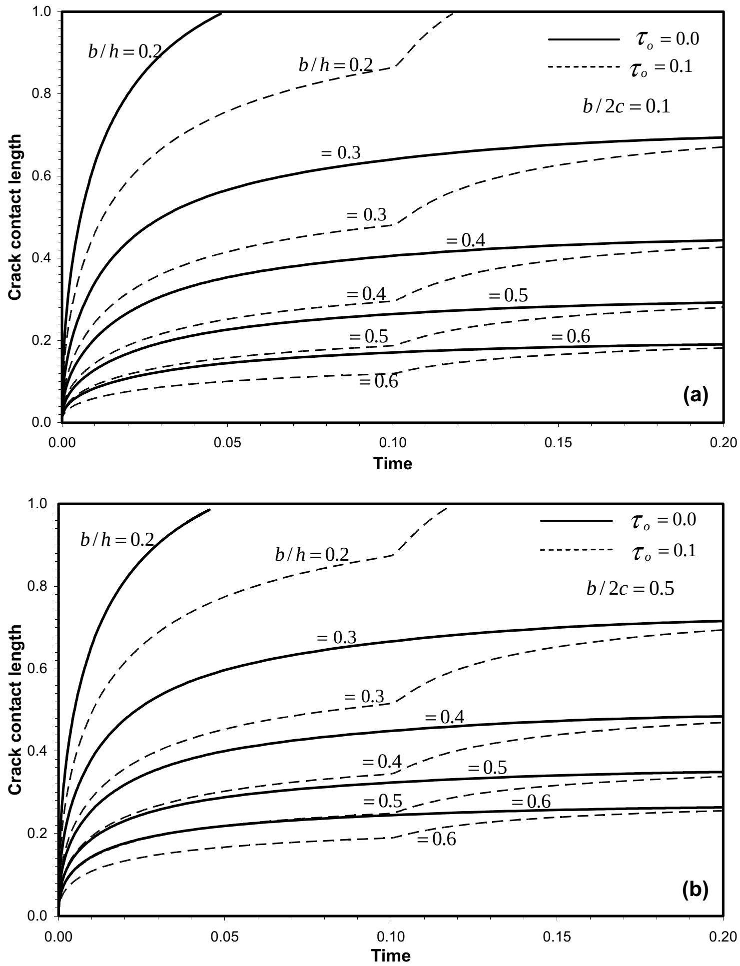

Fig. 6. Normalized crack contact length ε/b for different values of normalized crack length b/h. (a) $b/2c = 0.1$, (b) $b/2c = 0.5$.

intensity factors. The maximum magnitude of the normalized stress intensity factors starts to increase as the normalized crack length increases until a certain value and then it decreases.

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